Predicting Wave Runup on Dam Slopes

Introduction

When wind blows over an open water surface, such as within a reservoir, wind-generated waves can strike the upstream slope of the dam embankment. This can cause erosion of the embankment material and if severe enough, waves can overtop the embankment, both of which are dam safety issues. Therefore, the dam embankment design must consider the potential effects of wave action and protect against erosion of the embankment materials and overtopping due to wave runup. This is done by extending the embankment up from the still water flood pool level to an elevation equal to the still water pool plus the maximum calculated wave runup and wind setup height.

This article describes a procedure for calculating the wind-generated wave characteristics for inland reservoirs and lakes and the resulting wave runup on a sloping dam embankment for small dams.

Dominant Factors and Procedure

The major variables used to calculate wind-generated wave height on open water surfaces, such as reservoirs, and influence embankment design are:

- Effective Fetch and Wind Direction
- Wind Speed over Water
- Wind Setup, Wave Height and Runup

The procedure presented in this article is based on information presented in TR-69 (USDA, 1983) and Bureau of Reclamation ACER TM-No. 2. Additional information related to US Army Corps of Engineers (USACE) procedures is presented in the reference documents included at the end of this article.

Effective Fetch and Design Wind Direction

The procedure in this article is limited to reservoirs where 1.) Effective fetch is less than 10 miles and 2.) Wave height is less than 5 feet.

The fetch is an overwater length blown on at a constant wind speed and direction. The longer the fetch and the faster the wind speed, the more wind energy is imparted to the water surface and proportionally higher waves will be produced. TR-69 recommends two approaches to determine the design fetch and wind direction: (1) U.S. Weather Service climatological data or (2) site orientation. Because most dams/reservoirs are ungauged, wind data does not typically exist and the site orientation method is preferred to define the effective fetch and design wind direction.

The design wind direction is obtained by determining the longest stretch of open water from a point on the shoreline opposite to the dam embankment. It is assumed that wind and waves are developed along the longest fetch of open water from the dam. According to Saville's 1954 study, the width of the fetch on inland reservoirs normally places a definite restriction on the length of effective fetch, which is the effective distance of the water over which the wind blows without appreciable change in direction. **Figure 1** diagrammatically shows the central (longest) and radial fetch lines for a hypothetical reservoir. Simplistically, this method involves drawing the central radial line and then drawing seven radial lines at 6-degree intervals on each side of the central radial line.

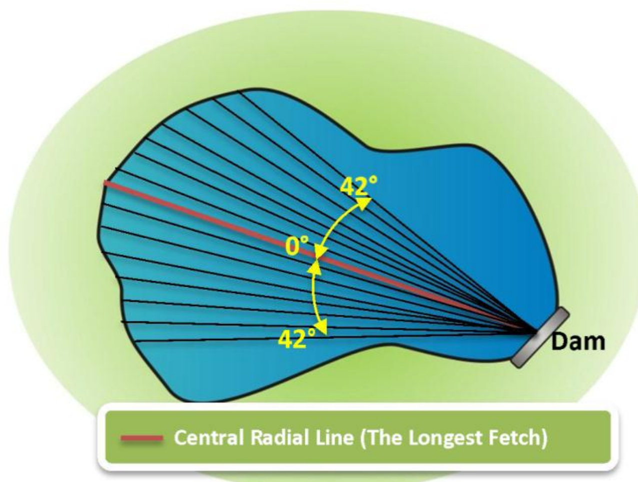

Figure 1: Central and radial fetch lines

The effective fetch, F_e, can then be computed using **Equation 1**.

$$F_e = \frac{\sum_{i=1}^{15}(X_i \cdot \cos^2 \alpha_i)}{\sum_{i=1}^{15}(\cos \alpha_i)} \qquad \text{Eq.1}$$

x_i = Length of Radial Line i

α_i = Angle Degree between the Central Radial Line and the Radial Line i

Wind Speed over Water

There are two common procedures for determining the design wind speed. They are:

1. *A constant overwater wind speed of 100 mph (Reclamation, 1987)*
2. *Site-specific wind speed and duration curves*

The 100 mph wind speed recommended by Reclamation is a simple but conservative approach. The more detailed site-specific approach is presented in the following paragraphs.

According to the guidelines titled Reclamation ACER TM-No. 2 and TR-69, the design wind speed and duration can be selected by using the observed maximum wind speed and the effective fetch. Commonly, the observed fastest mile wind speed is considered as the maximum overland wind speed, U_L, and can be obtained from the National Oceanic and Atmospheric Administration (NOAA) National Climatic Data Center websites shown at the end of this article. The NOAA wind data, including wind speed, duration, and direction, indicates the overland wind characteristics at 25 feet above ground.

The duration of a given wind speed that needs to be maintained to fully develop the maximum waves is a function of the effective fetch. The longer the effective fetch, the longer the duration for the sustained wind speed. **Figure 2** graphically shows the selection of design wind speed based on the relationship between the maximum wind speed and the effective fetch response to wind speed. The intersection of the red curve and blue curve identifies the "design wind speed."

The red line on **Figure 2** can be developed using the observed fastest mile wind speed and the information contained in Figure 5 of TR-69. Alternatively, **Table 1** is provided as a simplification of the information shown in Figure 5 of TR-69.

Careful!! The units for effective fetch and wind speed vary for the various equations in this article. Make note of units required for each eqn.

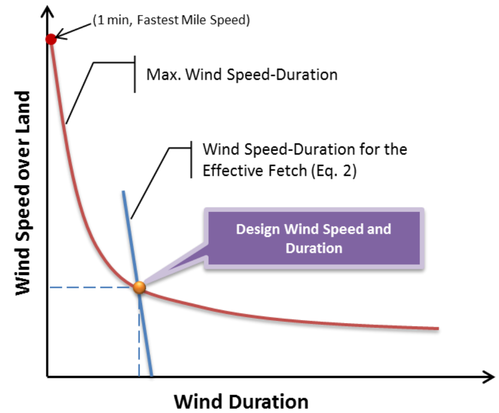

Figure 2: Plot of wind speed vs. duration

Table 1: Maximum wind speed relationship

Fastest Mile Wind Speed, mph	Ratio of Land Wind Speed to the Fastest Mile Wind for the Durations			
	1 min*	**30 min**	**60 min**	**100 min**
100	100%	52%	46%	41%
80	100%	57%	51%	47%
60	100%	65%	59%	55%

* Duration of fastest mile wind speed is one minute.

The blue curve in **Figure 2** needs to be generated using Figure 2 in TR-69 or the empirical relationship (**Equation 2**) of overland wind speed and duration for the site specific effective fetch.

$$g\frac{T}{U_L} = 27.99\left(\frac{g \cdot F_e}{{U_L}^2}\right)^{0.72} \qquad \text{Eq.2}$$

g = Gravitational Acceleration, 32.2 ft/sec^2
T = Wave Duration in seconds. Wave duration is equal to the minimum wind duration required for generation of wave heights for a specific effective fetch and wind speed.
U_L = Overland Wind Speed in ft/sec
F_e = Site Specific Effective Fetch in ft

Because of smoother and more uniform surface conditions, overwater wind speeds, U_w, are higher than overland wind speeds, U_L. To consider this speed enhancement, the overwater wind speed can be computed using the following equation.

$$U_w = \beta \cdot U_L \qquad \text{Eq.3}$$

β = Wind Speed Adjustment Factor or Ratio, U_w/U_L, Shown on **Figure 3**.

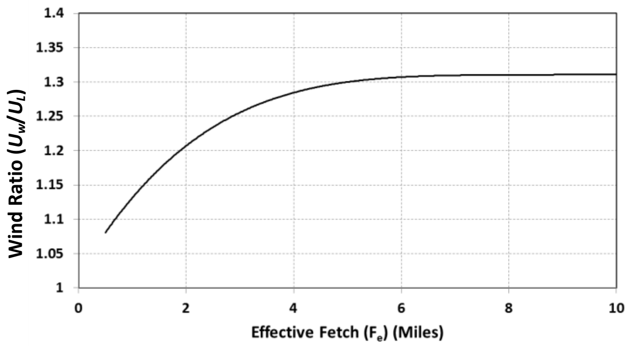

Figure 3: Wind speed relationship – water to land

Wind Setup, Wave Height, and Runup

A sketch of waves striking an embankment slope is illustrated in **Figure 4**. When wind is blowing over a water surface, horizontal shear stress acts on the water surface, and the water surface is tilted in the direction of the wind. This wind effect is termed "wind setup" and can be estimated using the empirical equation from TR-69 shown below.

$$S = \frac{{U_W}^2 \cdot F}{1400 \cdot D} \qquad \text{Eq. 4}$$

S = Wind Setup in feet
U_w = Wind Speed in miles per hour
F = Wind Fetch in miles (Approximately equal to F_e)
D = An approximation of the average water depth along the fetch length in feet

Slope protection is generally designed for what is known as the "significant wave height." The significant wave height is the average height of the highest one-third of the wind-generated waves. This means that 33 percent of the waves that hit the slope will be higher than this value. Based on the selected design overwater wind speed and the effective fetch, the significant wave height, H_s, and wave length, L, can be estimated using the following dimensionless equations from TR-69.

$$g\frac{H_S}{{U_L}^2} = 0.0026\left(\frac{g \cdot F_e}{{U_L}^2}\right)^{0.47} \qquad \text{Eq. 5}$$

$$g\frac{\sqrt{L}}{U_L} = 1.041\left(\frac{g \cdot F_e}{{U_L}^2}\right)^{0.28} \qquad \text{Eq. 6}$$

g = Gravitational Acceleration, 32.2 ft/sec^2
H_s = Significant Wave Height in feet
L = Wave Length in feet
U_L = Overland Wind Speed in ft/sec
F_e = Effective Fetch in feet

Equations 5 and **6** are empirical equations developed from deep-water waves, which are defined as waves having lengths equal to or less than $2D$. They also give conservative wave height estimations for shallow-water waves.

The significant wave height defined above would be exceeded by approximately 33 percent of the expected waves generated by the associated wind speed. If a lower potential of exceedance is desired, a wave height of 1.27H_s and 1.67H_s have a corresponding potential for exceedance of 10 percent and 1 percent, respectively.

When waves reach a sloping embankment, the waves will eventually break on the slope and run up to a height governed by the angle of the slope, and the surface roughness and permeability. Wave runup height, R, is the difference between the maximum elevation reached by wave runup on a slope and the storm water level. The steeper the embankment slope the greater the wave runup height. Many studies have been published that provide guidance for determining the wave runup height on slopes. The runup from a significant wave on an embankment slope with riprap protection can be predicted using:

$$R = \frac{H_S}{0.4 + \left(\frac{H_S}{L}\right)^{0.5} \cdot \cot\theta} \qquad \text{Eq. 7}$$

R = Wave Runup Height in feet
H_s = Significant Wave Height in feet
L = Wave Length in feet
θ = Angle of the Dam Face from Horizontal

Equation 7 should be used only for embankment slopes steeper than 5H:1V.

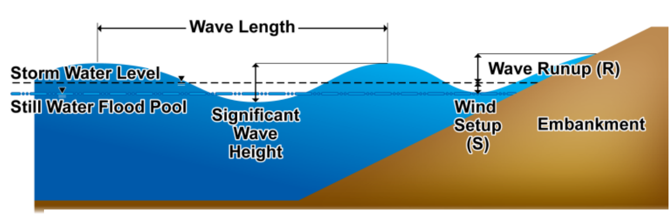

Figure 4: Sketch illustrating wave terms

Conclusions

The wind-generated wave characteristics and the related wind setup and wave runup on a sloping embankment within a reservoir must be considered for the purposes of designing embankments and embankment slope protection. Slope protection for the embankment must also be considered and a procedure for the design of riprap slope protection is described in the following article titled, "Design of Riprap for Slope Protection against Wave Action."

NOAA Climatological Data Links

Local Climatological Data:
http://www.ncdc.noaa.gov/IPS/lcd/lcd.html

Climate Maps of the United States:
http://cdo.ncdc.noaa.gov/cgi-bin/climaps/climaps.pl

NOAA Climate Data Online:
http://www.ncdc.noaa.gov/cdo-web/

References (with Links where available)

- USDA (1983), Technical Release No. 69: "Riprap for Slope Protection against Wave Action."
- Reclamation (1992), ACER Technical Memorandum No. 2: "Freeboard Criteria and Guidelines for Computing Freeboard Allowances for Storage Dams."
- Reclamation (1987), Design of Small Dams, Third Edition.
- USACE (1976), Engineering Technical Letter No. 1110-2-221: "Wave Runup and Wind Setup on Reservoir Embankments."
- Saville, Thorndike J. (1954), "The Effect of Fetch Width on Wave Generation," Technical Memorandum No. 70, Beach Erosion Board, USACE.

Example #1:

Find the wind setup, the wave height and the wave runup of a reservoir as shown on **Figure 1.** The observed fastest mile wind speed is 75 mph for this site. The average depth of the reservoir is 10 feet, and the riprap protected embankment has a 3H:1V or 18° slope.

Calculations:

1. To measure the lengths of the central (longest) and radial lines as shown in **Figure 1**, compute the effective fetch using **Equation 1.** The computation is shown in **Table 2**.

Table 2: Procedure to determine the effective fetch

Radial No.	Radial Length (mi), X_i	α (Degree)	cos α	$X_i \cdot \cos^2 \alpha$
1	1.7	42	0.74	0.96
2	1.8	36	0.81	1.20
3	1.9	30	0.87	1.45
4	2.0	24	0.91	1.70
5	2.2	18	0.95	2.02
6	2.3	12	0.98	2.23
7	2.4	6	0.99	2.41
8	2.6	0	1.00	2.63
9	2.5	6	0.99	2.51
10	2.4	12	0.98	2.33
11	2.3	18	0.95	2.11
12	2.1	24	0.91	1.78
13	2.0	30	0.87	1.53
14	1.8	36	0.81	1.20
15	1.7	42	0.74	0.96
		Sum=	13.51	27.02

$$F_e = \frac{\sum_{i=1}^{15}(X_i \cdot \cos^2\alpha_i)}{\sum_{i=1}^{15}(\cos\alpha_i)} = \frac{27.02}{13.51} = 2.0 \text{ miles}$$

This effective fetch of 2.0 miles or 10,560 feet from the given reservoir with a longest fetch of 2.6 miles is estimated.

2. Refer to Figure 5 of TR-69 or **Table 1** in this article, the generalized maximum wind speed-duration relationship is plotted as the red line on **Figure 5**. This is computed by using the observed fastest mile wind speed, 75 mph, interpolating the ratio of land wind speed to the fastest mile wind for each of the durations shown and then multiplying this ratio by the observed fastest wind speed. The results of these computations are shown in **Table 2**.

Table 2: Maximum Wind Speed-Duration Relationship for a Fastest Mile Wind of 75 mph

	1 min	30 min	60 min	100 min
Interpolated Ratio from **Table 1**	100%	59%	53%	49%
Corresponding Max. Wind Speed (mph)	75	44	40	37

3. By using **Equation 2** and the effective fetch, 2.0 miles, the relationship of overland wind speed-duration for the selected fetch is determined for a range of selected speeds (in this case, U_L= 90 mph, 60 mph, and 35 mph). Remember to first convert U_L to ft/sec and fetch length to feet. T is calculated in seconds with **Equation 2** and then converted to minutes for the plot. The results are shown as the blue curve in **Figure 5.**

$$32.2 \cdot \frac{T}{U_L} = 27.99 \left(\frac{32.2 \cdot 2.0 \cdot 5280}{U_L{}^2} \right)^{0.72}$$

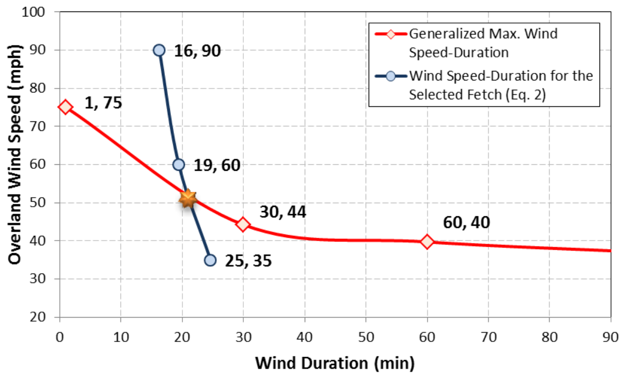

Figure 5: Plot of wind speed vs. duration

4. The intersection of the red curve and blue curve identifies the "Design Overland Wind Speed (U_L)" of 52 mph or 76 ft/sec. Find the overwater wind speed using **Figure 3** and **Equation 3**. This gives a wind ratio of 1.21 from the figure and an adjusted overwater wind speed (U_w) of 63 mph.

$$U_w = \beta \cdot U_L = 1.21 * 52\, mph = 63\, mph$$

5. Find the wind setup using **Equation 4**.

$$S = \frac{63^2 * 2.0}{1400 * 10} = 0.6\, feet$$

6. Find the wave height using **Equation 5**.

$$H_s = \frac{76^2}{32.2} \cdot 0.0026 \left(\frac{32.2 \cdot 10560}{76^2} \right)^{0.47} = 3.2\, ft$$

7. Find the wave length using **Equation 6**.

$$L = \left(\frac{76}{32.2} \cdot 1.041 \left(\frac{32.2 \cdot 10560}{76^2} \right)^{0.28} \right)^2 = 59\, ft$$

8. Find the wave runup height using **Equation 7**.

$$R = \frac{3.2}{0.4 + \left(\frac{3.2}{59} \right)^{0.5} \cdot \cot(18^0)} = 2.9\, ft$$

Results:

The estimated maximum significant wave height is 3.2 feet with an overwater wind speed of 63 mph. The corresponding maximum height the water will reach from the still water flood level is 3.5 feet, which is the sum of wind setup (0.6 foot) and runup (2.9 feet).